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ABSTRACT

The cognitive determinants of number series completion performance were studied by presenting a systematic set of problems to adults and to children of high and average intelligence, grades 4 and 5. Pearson correlations between these problems and the number series on the Cognitive Abilities Test were .51, .78 and .43 for the average intelligence, high intelligence, and adult groups, respectively. In each group, a combination of content and procedural knowledge factors accounted for 75% of the variance in item solution difficulty. The content factors involved understanding the sequential nature of numbers, and recognizing and executing arithmetic operations. Procedural factors were routines that established the pattern of arithmetic operation in sequence, for memory storage and recall. The most critical processing demand affecting successful performance was the amount of solution-related information to be assembled and managed in working memory. The content and procedural knowledge factors differentiated adults from children, while only content knowledge seemed to differentiate among IQ levels in children. (Author/CP)

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COGNITIVE DETERMINANTS OF SERIES COMPLETION: INDIVIDUAL AND DEVELOPMENTAL DIFFERENCES

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Cognitive Determinants of Series Completion: Individual and Developmental Differences

The present study is an attempt to gain information about the cognitive processes that underlie performance on standardized intelligence tests. This investigation focuses on a prominent type of inductive reasoning task, the number series completion problem (see Table 1 for examples). Inductive reasoning is of particular interest because factor analytic studies have frequently identified rule induction as an important factor emerging from the aptitude test performances of both adults and children (Thurstone and Thurstone, 1941; French, 1951; Adkins and Lyerly, 1952; Guilford, 1967; Cattell, 1971; Ekstrom, 1973). The regular occurrence of a rule induction factor in many test batteries suggests a close tie between inductive reasoning and school success, making this an especially important competency for educators to learn about. Furthermore, an investigation of inductive reasoning may yield some information about the psychological underpinnings of other intelligence factors as well, since Thurstone and Thurstone (1941) have asserted that rule induction, as a second order factor, may actually be Spearman's g. Series completion problems seem to be a particularly good context for investigating rule induction, because they constitute a major portion of some of the most widely used aptitude tests which incorporate a rule induction factor, including such batteries as the Cognitive Abilities Test (CAT) and the Primary Mental Abilities test (PMA).

Objectives

Three broad levels of issues will be addressed in this presentation. First, an effort will be made to identify some of the major solution requirements

imposed by different types of number series and to assess the influence of those requirements on rates of solution. This effort will be aided through the use of a well-defined theoretical framework that was originally proposed by Simon and Kotovsky (1963; Kotovsky and Simon, 1973) to explain adolescent and adult performance on letter series completion problems. A second goal of this study is the attainment of information about the way in which rule induction capabilities develop. This will be achieved by comparing the performances of subjects of different ages on distinct types of series to determine whether certain processing demands exert greater influences at one age level than another. Finally, an attempt will be made to identify the kinds of solution requirements that are most responsible for the individual differences in performance that accompany psychometrically devised intelligence tests. This will be accomplished by identifying specific solution requirements that discriminate between subjects of distinct mental ages within a single chronological age group.

Method

Materials

Since standardized tests frequently confound changes made along two or more problem dimensions and since there are usually only a few instances of any particular task characteristic, it is often difficult, if not impossible, to determine the independent contribution of each identifiable processing demand to performance. To circumvent these limitations, 90 number series completion items were especially developed for this experiment. These series incorporated and manipulated the same features that a preliminary study had shown to be characteristic of series on the CAT and PMA. However, the

experimentally devised problem set was more systematic in its manipulation of task characteristics and contained more instances of those characteristics for greater reliability in revealing sources of solution difficulty. Included among the task variables explored in the experimental items were the qualitative categories of arithmetic operations required for solution (e.g., addition, subtraction, etc.), the magnitudes of the operations (e.g., -1, -8, etc.), the period lengths of the series (i.e., the regular interval at which operations repeat themselves), the number of transformations that were utilized within a given period of the sequence, and the total length of the presented sequence.

Subjects

To permit the exploration of sources of developmental differences, the experimental number series were administered to subjects at two different age levels, fourth and fifth grade children and adult college students. To allow further insights into the nature of individual differences in intelligence, the fourth and fifth grade subjects were selected into two distinct groups on the basis of previously obtained IQ scores, an average intelligence group, with IQ's ranging from 86 to 103, and a high intelligence group, with IQ's ranging from 130 to 158. Altogether then, differences in age and general intelligence were represented by three groups, and 18 subjects were assigned to each of them.

Procedure

Prior to the administration of the experimentally devised number series, subjects attempted all grade levels of the CAT number series in a group setting, using the standard paper-and-pencil format and time limits prescribed by

the test manual. This test provided an opportunity for a subsequent check on whether the experimentally devised items were indeed tapping the same skills as the standardized test items that they were purportedly modeled after. If the total scores on the experimental instrument positively correlate with those on its standardized counterpart, this would support the applicability of the experimental test results to the standardized types of tests that we ultimately wish to understand.

In separate sessions following the administration of the CAT, subjects were individually tested on the experimental number series. One item was presented at a time on the screen of a CRT computer terminal, and the subject was allotted 45 seconds to type one number into the computer to extrapolate the sequence.

Results

All 54 subjects completed the CAT number series and the experimentally devised series. The correlations between the scores on these two sets of items indicated that the experimental test was fulfilling its intended purpose in systematically tapping competencies underlying standardized test performances. For the average IQ children, high IQ children, and adults, respectively, the Pearson correlations between total scores on the two instruments were .51($p < .05$), .78($p < .001$), and .43($p < .10$).

Analyses of subjects' errors on individual items of the experimental test showed that two general classes of solution requirements contributed to problem difficulty. These can be referred to as domain-specific knowledge demands and general procedural knowledge demands. This dual set of number series solution influences parallels earlier suggestions by Simon and Kotovsky

that letter series performance relies on recognition of specific alphabetic relations and on higher order procedures that establish the pattern of those relations. For number series, the needed domain-specific knowledge entails a general understanding of the sequential nature of numbers, as well as skill in recognizing and executing specific arithmetic operations (e.g., addition by two, division by three, etc.). On the other hand, the general procedural knowledge involves several routines that establish the pattern of arithmetic operations in the sequence. These routines include the discovery of the periodicity of the series, the assembly of a solution rule that specifies the pattern of relations within and between periods, the management of solution-related information in working memory, and the application of the solution rule to extrapolate the sequence.

Both the domain-specific demands and the procedural knowledge demands of the number series were incorporated as predictor variables in multiple regression analyses designed to account for variations in the probability of correctly solving individual problems. These regressions accounted for 77% of the variance in solution rates for the average IQ children, 75% for the high IQ children, and 70% for the adults (all p's < .001).

Working Memory Demand

Of the different predictors included in the regression analyses, the most powerful one for all three groups of subjects was a general procedural knowledge requirement, the working memory load imposed by an item. Series vary in the number of numerical operations that they incorporate, and the more operations that must be assembled and kept track of, the more mental work space is required. Simon and Kotovsky (1963; Kotovsky and Simon, 1973) have cited

working memory load as a critical variable in the solution of letter series, and they have developed a notational scheme to exhibit the structure of series patterns, including their memory demands. This notational system was adapted to the number series task in order to make possible direct comparisons of the solution requirements imposed by different items. For example, Table 1 shows that the first series can be fully described by a constant, since no arithmetic transformations are needed for the solution of that problem. On the other hand, the second item requires that the examinee keep track of consecutive occurrences of an addition operation. This involves outputting the current value of a place-keeper in working memory (M_1) and subsequently replacing the value of that place-keeper with a number that is one larger ($+1(M_1)$). Item 7 imposes the greatest memory demands of any of the problems in Table 1. To solve that problem, three distinct arithmetic operations must be applied at three-number intervals, division by two for the subseries that begins with the first number (notated as M_1, $\div 2(M_1)$), division by three for the subseries that begins with the second number (M_2, $\div 3(M_2)$), and division by four for the subseries that begins with the third number (M_3, $\div 4(M_3)$). Apparently, increases in solution-related information quickly overload the limited mental work space that most people possess and constitute the most critical determinant of item difficulty for this type of task. The consistency of the results for the current number series task with those obtained by Simon and Kotovsky for letter series indicates that the importance of information coordination is not restricted to rule induction problems in one content domain, but extends at least across domains consisting of ordered sets of elements.

In order to explore sources of developmental and individual differences in the series, a two-way analysis of variance was applied to the data. Item features and subject characteristics were the independent variables, and solution rates in individual items again constituted the dependent variable. As expected, adults solved significantly more problems than children did (85% versus 52.5%, $p < .001$) and high IQ children solved significantly more than average IQ children (58% versus 47%, $p < .05$). Moreover, the age of the subjects interacted with the memory demands of the items ($p < .001$). As can be seen, in Figure 1, the children's performance began to deteriorate as soon as any increases were made in information coordination demands beyond the repetition of a single integer. The adults, on the other hand, performed with virtually the same accuracy over the first three information coordination levels, but demonstrated a decrement in performance at the fourth, and most demanding, level. This pattern of interaction effects is consistent with research by Pascual-Leone (1970) and Case (1972, 1978) suggesting that the effective capacity of a person's information management space, or working memory, increases developmentally. Apparently, the adults in this study had evolved greater amounts of storage space or better strategies for using that space which enabled them to effectively manage more rule-related information than their younger counterparts. Eventually, however, even those expanded capabilities became insufficient as the highest memory level problems were encountered.

Domain-Specific Knowledge Demands

In addition to the effects of working memory, the multiple regression analyses and the analysis of variance indicated that solution rates were

affected by some of the domain-specific demands of the series. Even among items requiring a single place-keeper in memory, there was variation in performance, and this variation seems to have been linked to differences in the types of simple arithmetic operations that the problems incorporated. Specifically, when single place-keeper series were divided into two groups on the basis of the relative magnitudes of the arithmetic operations they employed (e.g., Item 2 versus Item 4 in Table 1), comparisons of the two sets indicated that the high magnitude series were considerably more difficult than their low magnitude counterparts ($p < .001$). While the two groups of children were seriously impaired by high magnitude operations, the performance of the adults was virtually unaffected by this task dimension. (see Figure 2).

Another domain-specific variable that influenced performance was the qualitative category of the arithmetic operation governing the numbers of the series. As illustrated in Figure 3, the effects of this characteristic interacted with the age and the intelligence level of the subjects (p's $< .001$). The adults were so well practiced in arithmetic that the majority of their solutions were correct, regardless of whether an addition, subtraction, multiplication, or division operation was required to complete the sequence. On the other hand, the children experienced considerably more difficulty with multiplication and division than with addition and subtraction, and this effect was more pronounced for the average IQ children than the high IQ children. These domain-specific effects are especially notable, because they reveal that individual and developmental differences on tests of abstract reasoning may be due to differences in factual knowledge directly taught in school in addition to differences in the ability to assemble and reason with these facts in establishing the pattern of a problem display.

Discussion

The research reported here suggests that experimental analyses of psychometrically devised tests can lend psychological construct validity to instruments already valued for their predictive capabilities. The cognitive determinants of number and letter series performances seem to fall into two major categories, solution components directly tied to a particular content knowledge domain and higher order procedural components that detect regularities in the relationships revealed by the domain-specific skills. The superior scores of older examinees derive from both of these classes of solution capabilities, while performance differences between children of distinct intelligence level seem to stem primarily from differences in domain-specific knowledge. The challenge for future research is to better specify the mechanisms associated with the activation and growth of these different solution determinants and to assess the utility of this general theoretical framework for explaining performance on a wider variety of rule induction tasks.

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Table 1

Examples of Number Series and Pattern Description Notation

Problem	Pattern Description
1) No Numerical Transformations (Identity Relation)	
64 64 64 64 64 64 64 . . .	[64]
2) One Low Magnitude Addition Operation (+1)	
12 13 14 15 16 . . .	[M_1, + 1(M_1)]
3) One Low Magnitude Subtraction Operation (-1)	
20 19 18 17 16 . . .	[M_1, - 1(M_1)]
4) One High Magnitude Subtraction Operation (-8)	
43 35 27 19 11 . . .	[M_1, - 8(M_1)]
5) One Low Magnitude Multiplication Operation (x2)	
3 6 12 . . .	[M_1, x 2(M_1)]
6) Two Low Magnitude Subtraction Operations (-1, -3)	
32 21 31 18 30 15 . . .	[M_1, - 1(M_1), M_2 - 3(M_2)]
7) Three Low Magnitude Division Operations (÷2, ÷3, ÷4)	
24 81 32 12 27 8 6 9 2 . . .	[M_1, ÷ 2(M_1), M_2, ÷ 3(M_2), M_3, ÷ 4(M_3)]

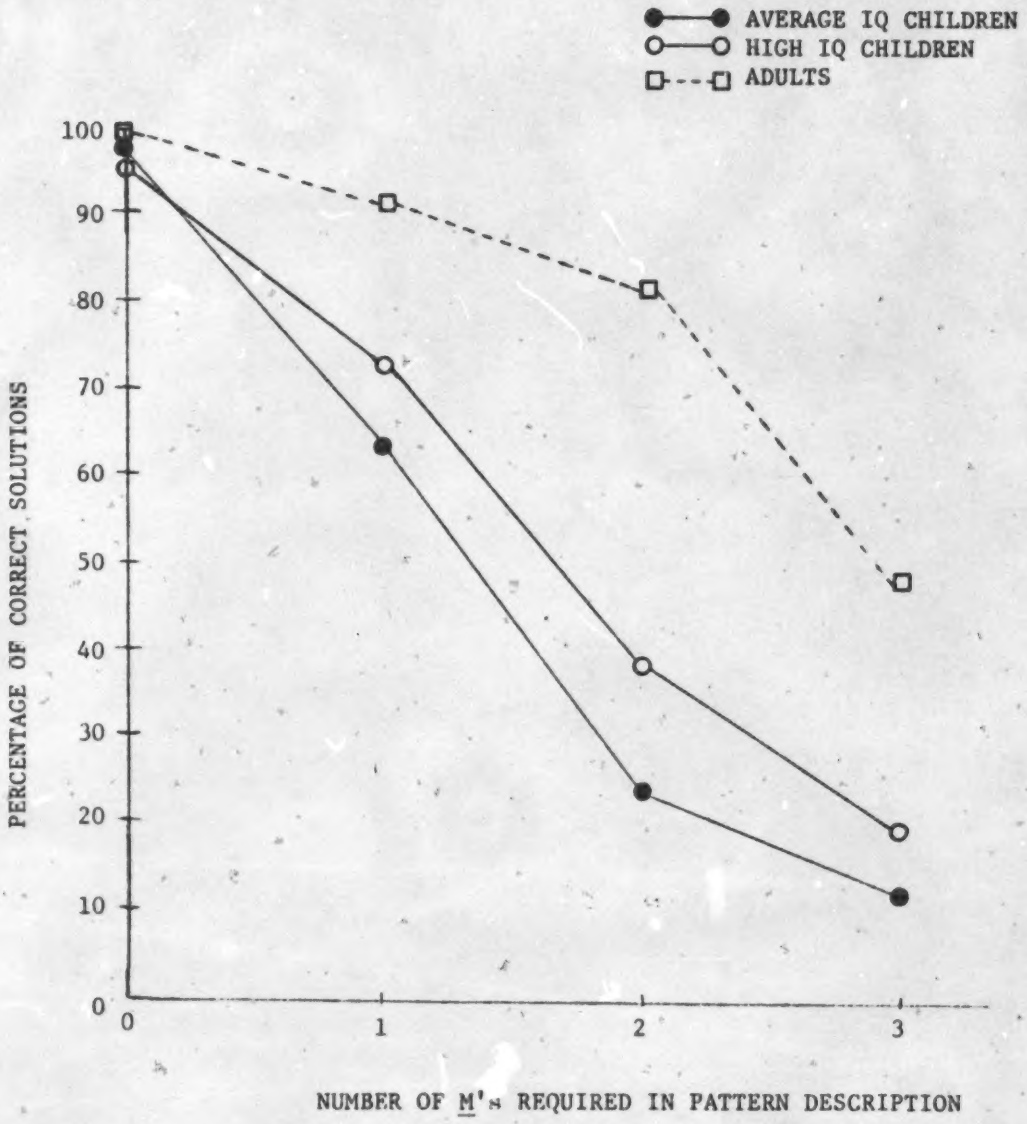

Figure 1. Percentages of correct solutions for number series with varying working memory requirements.

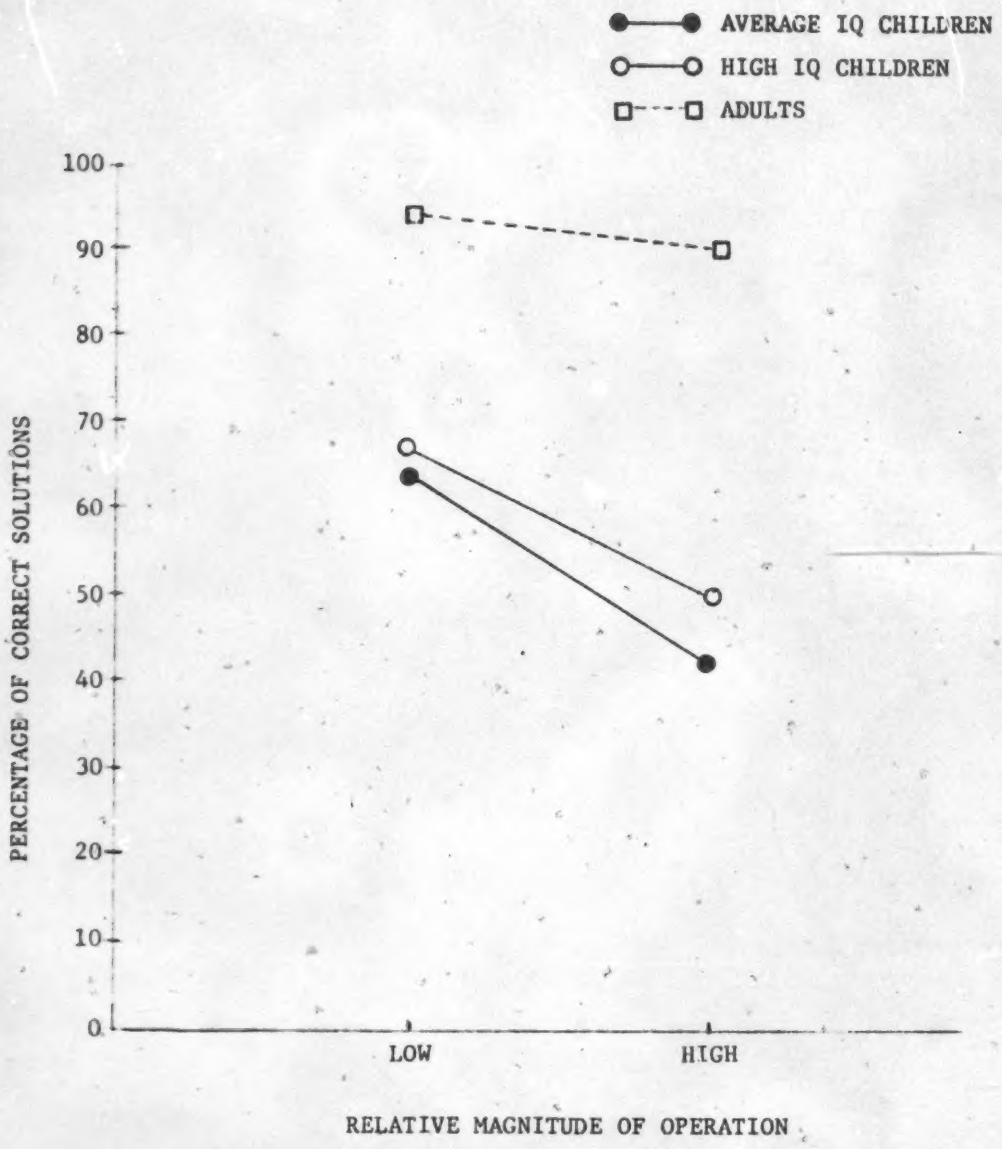

Figure 2. Percentages of correct solutions for number series requiring different magnitudes of arithmetic operations.

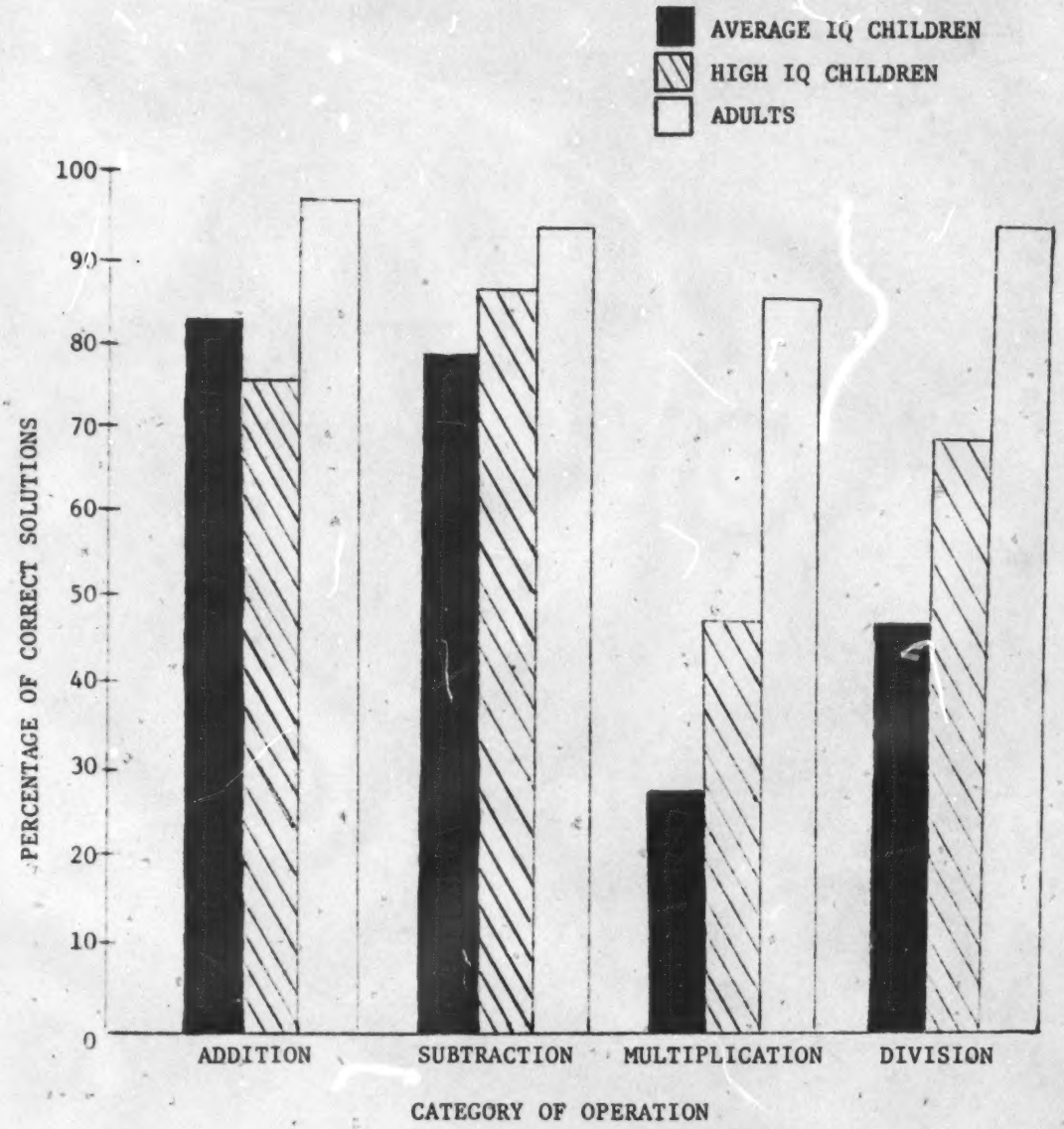

Figure 3. Percentages of correct solutions for number series requiring different qualitative categories of operations.